Robust 3D Tomographic Imaging of the Ionospheric Electron Density

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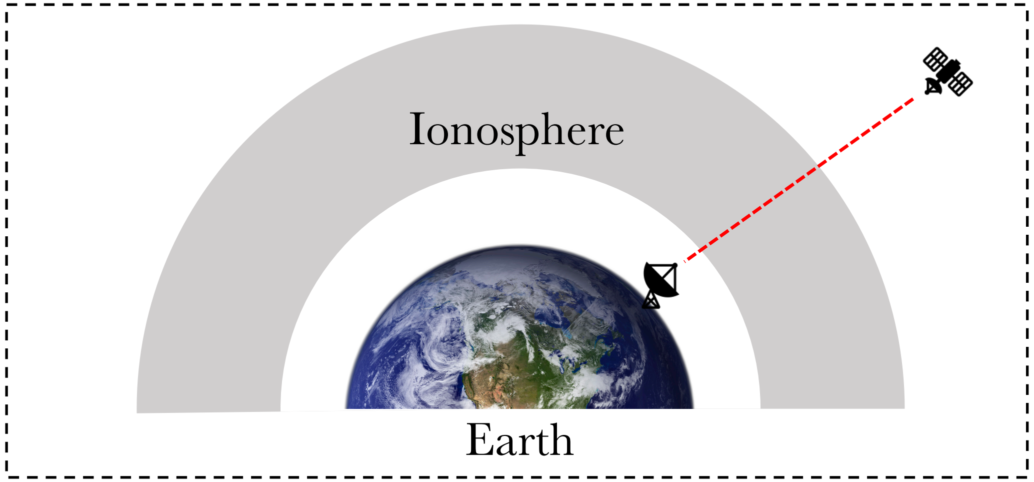
Background

- The ionosphere is the ionized region of the Earth's atmosphere spanning the altitudes between 60km to 1000km above the Earth's surface.



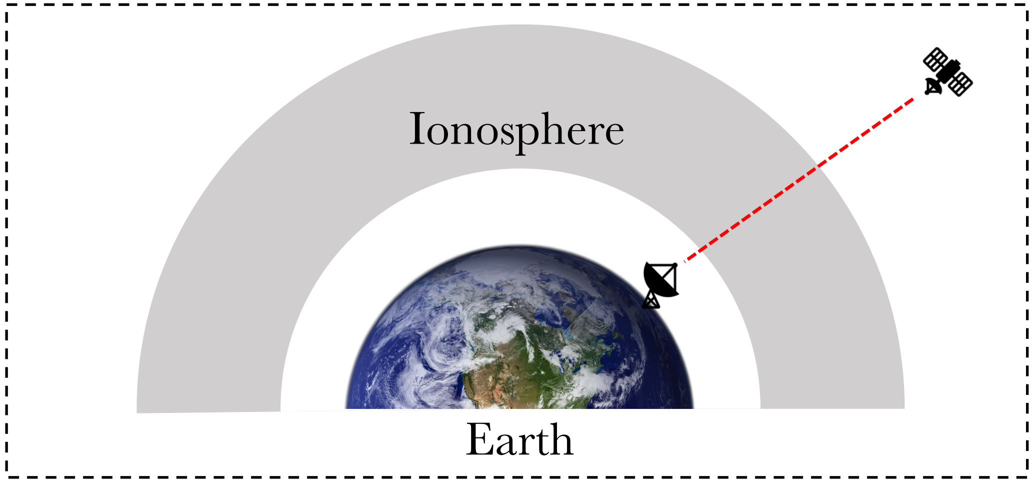
- The electrons act as a transportation medium as well as an interference channel for electromagnetic signals that are utilized by the global positioning system (GPS).

- Objective: Estimate the electron density distribution.

3D Tomographic Imaging Model

- Fact: Detectors only record the total electron content (TEC) along the line-of-sight (LOS)

$$\text{TEC} = \int_{rec}^{sat} N_e(\rho)d\rho,$$



where

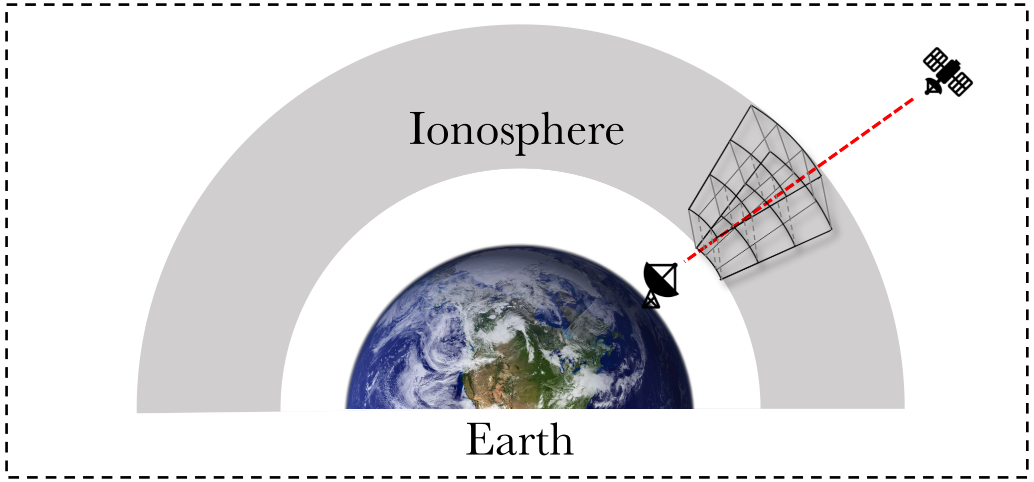
- TEC is the total electron content.
- $N_e(\rho)$ denotes the electron density along the ray path connecting the receiver and satellite.

R. Austen et.al., "Application of computerized tomography techniques to ionospheric research," in International Beacon Satellite Symposium, 1986, pp. 25–35
Jeffrey R Austen et.al., "Ionospheric imaging using computerized tomography," Radio Science, vol. 23, no. 3, pp. 299–307, 1988

3D Tomographic Imaging Model

- **Discretization :** Divide the three-dimensional space into small grids and the approximate(TEC) given by

$$\text{TEC} = \sum_{k=1}^{n} a_k x_k,$$



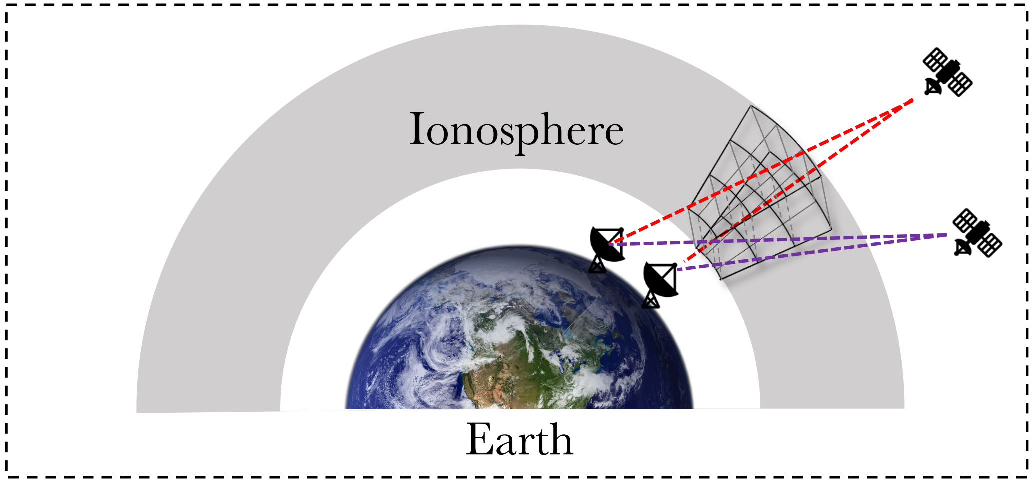
where

- k is the total number of gridded boxes.
- a_k denotes the length of the path in grid k.
- x_k denotes the electron density in grid k.

3D Tomographic Imaging Model

- Discretization : Divide the three-dimensional space into small grids and the approximate(TEC) given by

$$y_i = \sum_{k=1}^{n} a_{ik} x_k$$



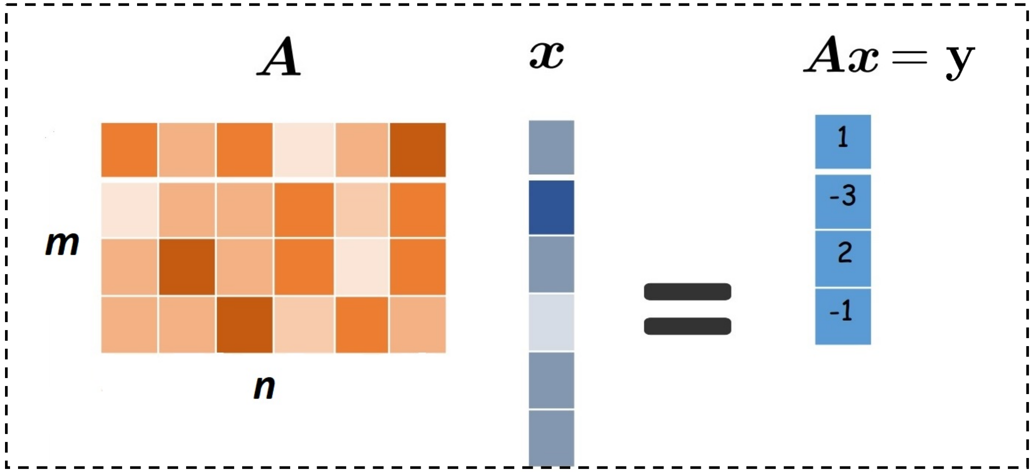
where

- k is the total number of gridded boxes.
- a_{ik} denotes the length of the path i in grid k.
- x_k denotes the electron density in grid k.

3D Tomographic Imaging Model

- 3D Model: Interpret the multiple linear combination as the matrix multiplication

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$



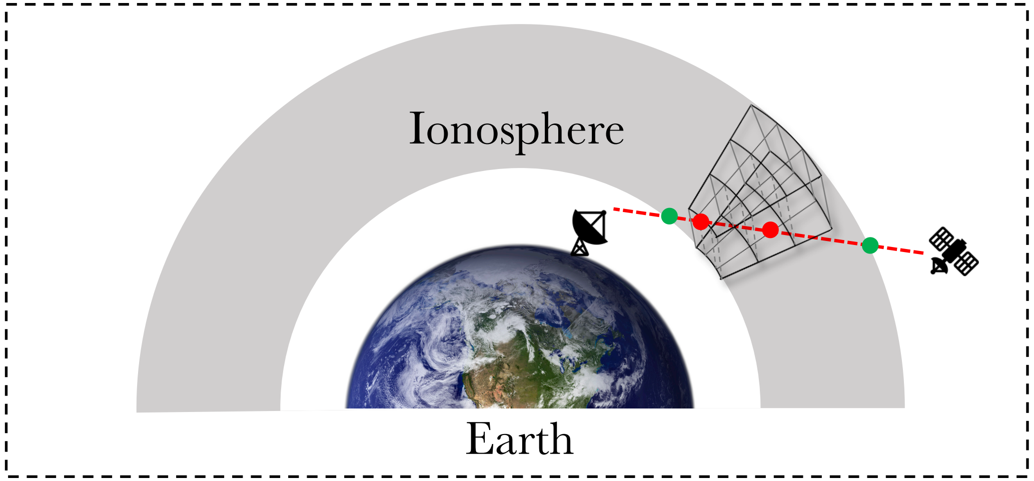
where

- m denotes the total number of satellite-receiver paths.
- n denotes the total number of grids.

G Ma and T Maruyama, "Derivation of tec and estimation of instrumental biases from geonet in japan," in Annales Geophysicae, 2003, vol. 21, pp. 2083–2093.

Compute the measurements

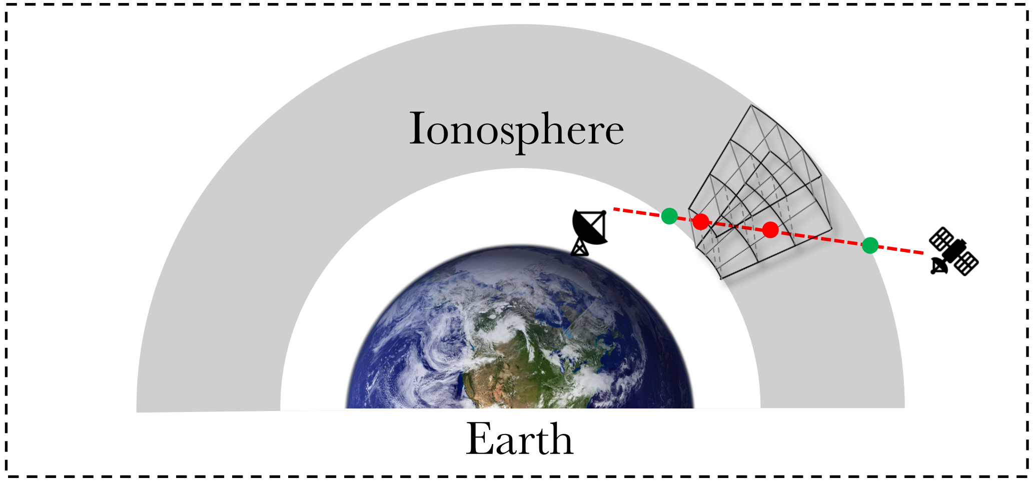
- Low angel measurements y_l : Need to discount the proportion of the TEC measurements that originate outside of the target domain.



Low angel measurements

Compute the measurements

- Low angel measurements y_l : Assume that the GPS-TEC inside the (region-of-interest) ROI along a LOS is proportional to the TEC inside the 3D grid of the ionospheric density estimated by the NeQuick model.



Low angel measurements

$$\tilde{y}_l = y_l \left(\frac{\text{partial TEC}_{l,\text{NeQuick}}}{\text{TEC}_{l,\text{NeQuick}}} \right)^p, \ \forall\, l \in \mathcal{L}$$

Y. Yao et.al., "A modified three-dimensional ionospheric tomography algorithm with side rays," GPS Solutions, vol. 22, no. 4, pp. 15211886, 2018.

Reconstruction of unknown electron density distribution

- Proposed method : We formulate the problem of reconstructing the ionospheric volume as the following regularized least squares problem:

$$\widehat{\mathbf{x}} = \operatorname*{arg\,min}_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{W}\mathbf{x}\|_2^2 + \gamma\sum_{q=1}^{h}\|\mathbf{R}_q\mathbf{x} - \mathbf{x}_q\|_2^2$$

$$\text{s.t.} \quad \mathbf{x} \geq 0$$

where

- $\mathbf{W} \in \mathbb{R}^{n\times n}$ is a constraint matrix.
- $\mathbf{x}_q$ is the electron density for a fixed latitude and longitude
- $\mathbf{R}_q$ is a binary selection matrix
- h is the number of reference points.
- $\lambda \geq 0$ and $\gamma \geq 0$ are the regularization parameters.

Reconstruction of unknown electron density distribution

- Proposed method : We formulate the problem of reconstructing the ionospheric volume as the following regularized least squares problem:

$$\widehat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^n}{\arg\min} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{W}\mathbf{x}\|_2^2 + \gamma \sum_{q=1}^{h} \|\mathbf{R}_q\mathbf{x} - \mathbf{x}_q\|_2^2$$

data-consistency

Reconstruction of unknown electron density distribution

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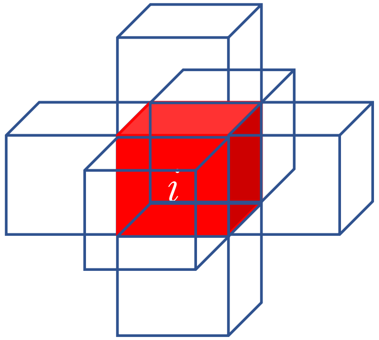
$$\widehat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^n}{\text{arg min}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{W}\mathbf{x}\|_2^2 + \gamma \sum_{q=1}^{h} \|\mathbf{R}_q\mathbf{x} - \mathbf{x}_q\|_2^2$$

coupling constrain

$$(\mathbf{W}\mathbf{x})_i = \sum_{k=1}^{6} C_{ik}(x_i - x_{ik})$$



where

- $C_{ik} \geq 0$ denotes the coupling of the electron density in grid i with the electron density in the six neighboring grids.
- $C_{ik} \geq 0$ are determined as a function of the latitude, longitude, and altitude based on the empirical electron density model NeQuick.

Reconstruction of unknown electron density distribution

- Proposed method : We formulate the problem of reconstructing the ionospheric volume as the following regularized least squares problem:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^n}{\arg\min} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{W}\mathbf{x}\|_2^2 + \gamma\sum_{q=1}^{h}\|\mathbf{R}_q\mathbf{x} - \mathbf{x}_q\|_2^2$$

reference constrain

Numerical results

- Reconstruction: We focus on the reconstruction of 3-D ionosphere density model in the region above Japan at 13:30 UT on May 17, 2019 with 500 GPS ground stations.

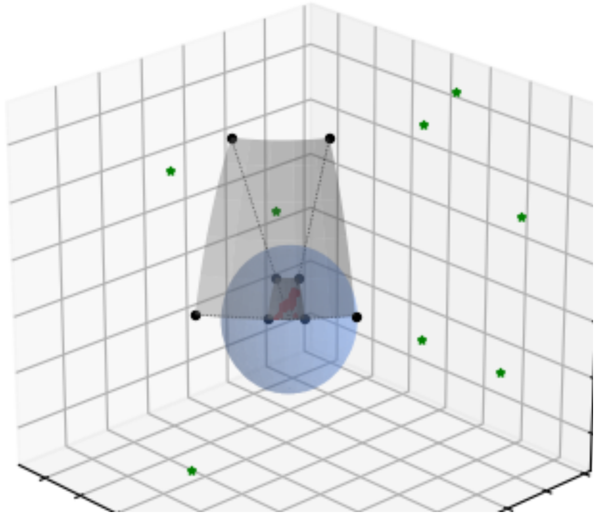


Illustration of the observed satellites and GPS ground stations in the region above Japan at 13:30 UT on May 17, 2019. The gray shaded region illustrates the reconstruction volume.

http://datahouse1.gsi.go.jp/terras/terras_english.html

Numerical results

- Ground truth x*: We conduct simulation-based experiments using the NeQuick model as ground truth x*.
- Forward model : We construct the forward operator A corresponding to the specified date and time.
- Measurements: We synthesize the TEC measurements by multiplying A with x*.

$$\text{relative error} = \|\widehat{\mathbf{x}} - \mathbf{x}^*\|_2 / \|\mathbf{x}^*\|_2$$

(RE)

Numerical results

- Robustness comparison: Our proposed approach, remains robust to model mismatch, whereas modified SIRT is more seriously affected by the measurement error.

$$\tilde{y}_l = y_l \left(\frac{\text{partial TEC}_{l,\text{NeQuick}}}{\text{TEC}_{l,\text{NeQuick}}} \right)^p, \ \forall\, l \in \mathcal{L}$$

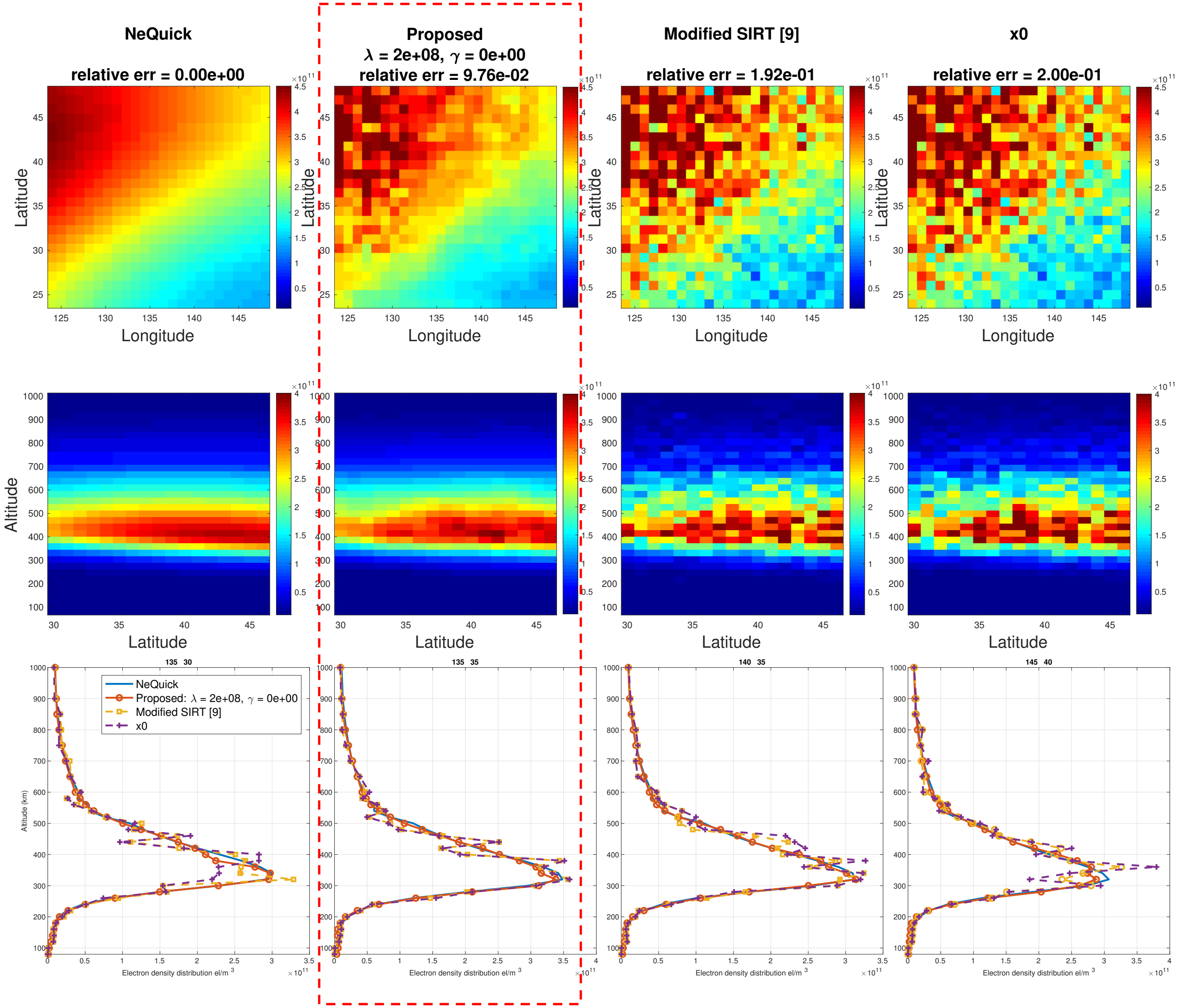
Table 1: Relative error (RE) sensitivity to mismatch in partial TEC

Exponent p	Proposed RE	modified SIRT RE
1	0.0976	0.192
2	0.104	0.197
4	0.133	0.215

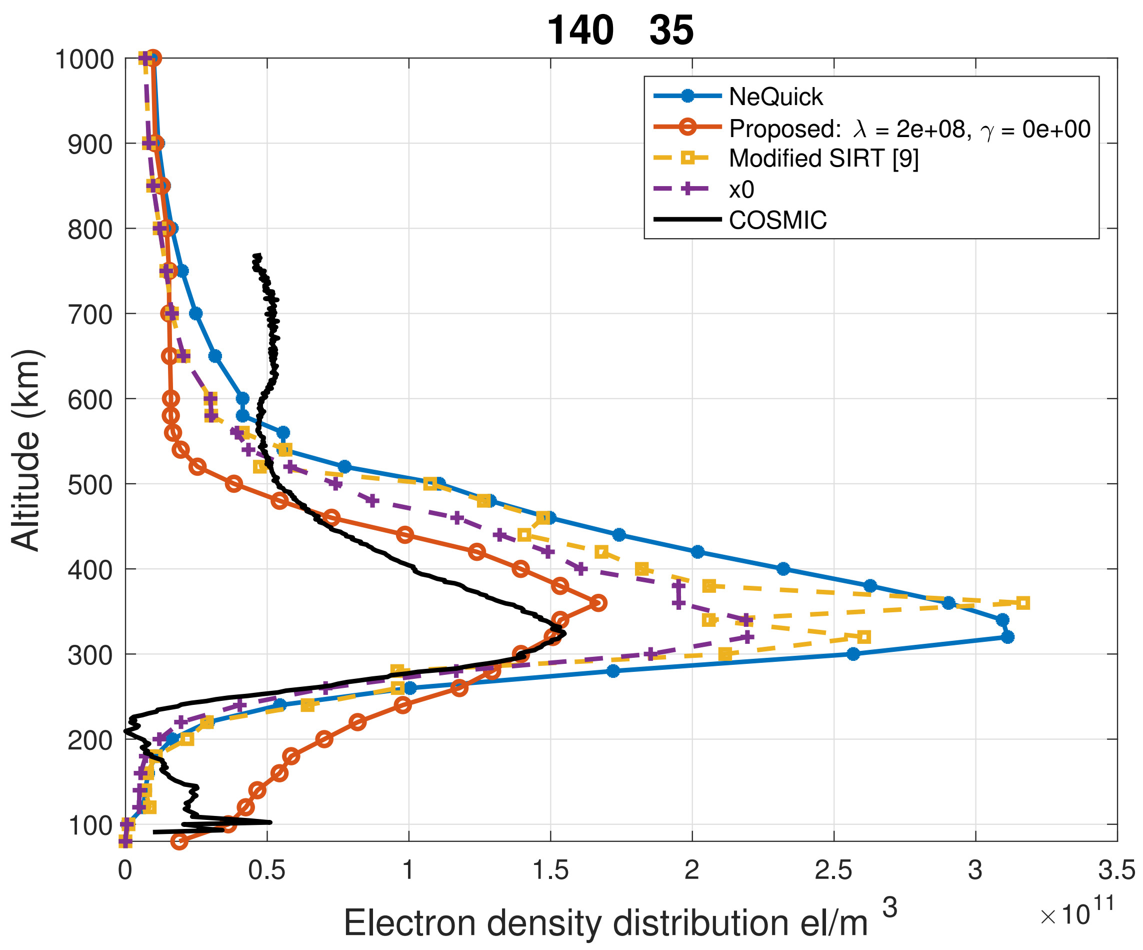
[9] Y. Yao, et.al., "A modified three-dimensional ionospheric tomography algorithm with side rays," GPS Solutions, vol. 22, no. 4, pp. 1521–1886, 2018.

Numerical results

Right: Comparison of the reconstruction performance from simulated TEC measurements with the modified SIRT method in [9]. The first row show the horizontal slice at elevation 300 km, the second row shows a meridional slice at longitude 135°E and the third row shows the vertical profiles at [135 ° E, 30◦ ° N], [135 ° E, 35 ° N], [140 ° E, 35 ° N], and [140 ° E, 40 ° N].



Numerical results



Above: Comparison of vertical electron density profiles from real data.

http://cdaac-www.cosmic.ucar.edu/

Conclusion

- We develop a robust 3D tomographic imaging framework to estimate the ionospheric electron density using ground-based total electron content (TEC) measurements from GPS receivers.
- We incorporate into the tomographic measurements the TEC readings observed from low-angle satellites that fall outside of the target ionospheric domain.
- We demonstrate through simulations that our framework delivers superior reconstruction of the ionospheric electron density compared to existing schemes. We also demonstrate the applicability of our approach on real TEC measurements.

Thanks!